# W. L. Li Ohio State University\*

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During the past few years there has been considerable interest in constructing mathematical models for the study of the educational process. These models, according to their subjects, can be classified into three categories (See Wurtele, 1967). The first category may be called demographic models. They focus primarily on the educational system or some of its components. such as the flow of students, teacher-student ratio, etc. The second category are the econometric models which treat education as one of the several interrelated economic activities; educational institutions are viewed as producers of outputs that are employed by the different sectors of society. The third category of models deals with the learning outcomes of individual students, or group of students. The sociopsychological aspects of educational process are strongly emphasized.

This paper is concerned with the first category of educational models. It attempts to examine student progression in an educational system from the demographic point of view. So, the subject of educational process is treated in the aggregate, and the interdependencies of the educational system with other sectors of society is analytically disregarded.

# A General Demographic Model

The subject of educational process has long been of great interest to demographers. And the demographic analysis of educational process has been a great contribution to the educational planners, who must continually estimate the size of future student enrollments at different levels of the educational structure. Examples are found in the work of the Census Bureau, which in the past years has provided a continuous projection of school enrollments (Census Bureau, 1963; Siegel, 1967). A systematic exposition of educational demography using the Census Bureau's statistics is shown in the publication of Folger and Nam (1967).

However, demographers are often blamed for their failure to make an accurate educational projection. It is sometimes complained that demographers have relied too much on the techniques of trend extrapolation. Besides, the reliability of school enrollment analysis seems to be dependent on the birth-death projection of the total population which itself may be inaccurate.

Let us begin with an examination of the general demographic methodology of projecting school population. It can be best summarized in the following equation (Stone, 1966): s = n • q

where s is the total student enrollment; n is the population vector of each age group; q is also a vector which gives the age-specific enrollment rates.

One way to implement the population vector is to follow Leslie's matrix approach, as Stone suggests. The Census Bureau's projection technique, though slightly different, nevertheless, is more or less based on trend extrapolation. Similarly, the enrollment rates are generally computed as a linear projection of the trend in observed fall enrollment rates in the past years.

One of the main limitations of such model, as Correa points out (Correa, 1967:34), is that the projected educational enrollments closely reflect the differences in the population structure and they are inappropriate to be used for temporal or spatial comparisons. There is another important limitation of such a model. It is that the model fails to give sufficient attention to the underlying dynamics of the educational process. This weakness is similar to the econometricians' construct of labor force function, which yields very limited knowledge of how the size of the labor force is determined by population structure.

## Structure of Educational System

In order to reach a more realistic projection of student population, we propose to begin with an examination of the underlying mechanism of educational process. The major part of this paper attempts to assess the underlying dynamics of cohort student progression. Let us first consider the following Lexis diagram which represents the progression of a student cohort (Fig. 1). $\underline{1}$ 

The example shows that the student cohort first enters the educational system in 1954. With the increment of years, its size changes from the first grade to the last grade. Symbolically the size of the cohort may be denoted as N(x,t), where x = 1, 2, ..., 12 and t = 1954, 55, ... It is obvious that the size tends to decrease over time in a population which is closed against immigration. If we take the initial size of the cohort as a basis, it can be shown (Fig. 2) that the decline of this cohort size is very much similar to a negative exponential distribution. It can be generalized as having the form of  $N(x) = N(1) \cdot e^{-kt}$ , where k is a constant term. For different cohorts, there will be different constant terms. It is possible to test empirically the variation of the terms for

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 $<sup>\</sup>underline{l}/$  An extensive application of Lexis diagram to demographic analysis can be found in Pressat (1961).



cohorts at different points in time; so some interesting results may be shown.

Nevertheless, examining only the behavior of the constant terms can be misleading. Obviously the curve is not a smooth one. A more realistic approach is to analyze a student cohort's "retention rate" (Duncan, 1965:129).2/ Symbolically it is N(x+1, t+1)/N(x,t). Fig. 2 indicates that the rate does not show any tendency of monotonous decline. On the contrary, it increases from the first grade to the sixth grade. Only after the eighth grade does the rate begin to decline monotonously. This observation leads to the conclusion that the decline of a student cohort is not strictly comparable to the survival curve in the general human population.

This suggests that a student population of a given grade is not only affected by those separating forces such as mortality and dropout. Many other factors can also contribute to determine the cohort size. Probably the most important factor is the effect of failure (or repeaters) at each level of an educational system. The effect is analogous to that of a rolling snowball; the repeaters tend to increase the rate of retention as shown in Fig. 2. Some other less important factors can be immigration and the reenrollment of dropouts.

Normally these two forces work in the opposite direction: mortality and dropout on the one hand, failure and new entry on the other hand. If the separating force is predominant, the size of a student cohort tends to decrease, and, hence, the retention rate will be less than unity. However, the effects of failure and new entry can be so strong that the decreasing tendency is cancelled out, which is obviously the case in Fig. 2. The combined effect of mortality and dropout is called the "separation factor" (Stockwell and Nam, 1963).

## An Educational Model from the Markov Process

Several simplifications must be made before we can construct a model which will take account of these underlying dynamics. We shall limit ourselves to the analysis of a closed population, an assumption which is not uncommon in demographic analysis. Furthermore, following a model of the Norwegian educational system (Thonstad, 1967), we assume that dropout is a one-time process; even though some dropouts re-enroll in schools in the later years, there are reasons to believe that the number may be too small to affect the total size of a school population. Tn other words, it is assumed that dropout is an absorbing state. Once an individual has left the educational system, he will not return.

Based on these assumptions, it follows that the progression of a student cohort has four alternatives, namely, failure, progress, death and dropout. The flow distribution of the cohort is a vector, and so the total educational system can be contrived as an input-output matrix, shown in Table 1.3/ The figures in the matrix are purely illustrative. The first three rows represent the educational system up to the third grade; their row sums show the number of students in a given grade; their elements give the flow distribution of those students in that grade. The last three rows represent graduation (g), mortality (m) and dropout (d), respectively. Their row sums are zero, as they are assumed to be absorbing states.

#### TABLE 1

## AN EXAMPLE OF STUDENT PROGRESSION

t2 t1	lst grade	2nd grade	3rd grade	Gradu- ation	Mor- tality	Drop- out	Sum
1st grade	75	415	0	0	5	5	500
2nd grade	0	45	391	0	5	9	450
3rd grade	0	0	20	360	4	16	400
Graduation	0	0	0	0	0	0	0
Mortality	0	0	0	0	0	0	0
Dropout	0	0	0	0	0	0	0
Sum	75	460	411	360	14	30	1350

It is obvious that such an input-output matrix can be readily formulated into a transition matrix, shown as follows:

	1	2	3	g	m	d
1	.15	.83	0	0	.01	.01
2	0	.10	.87	0	.01	.02
3	0	0	.05	•90	.01	.04
g	0	0	0	0	0	0
m	0	0	0	0	0	0
d	0	0	0	0	0	0

All elements in the matrix give the transitional probabilities from one state to another. Let us use  $A(i \cdot j)$  to denote such a transitional matrix. If the matrix is treated as stationary, it is possible to obtain the grade distribution of a student population at the successive years through an application of the elementary Markov process principle. It involves the use of the following equation:

(2) 
$$N_{(i)}^{t+1} = N_{(i)}^{t} \cdot A_{(i,j)}$$

where the grade distribution of students at time t+l is the result of multiplying the grade distribution of students at time t by the transitional matrix.

<sup>2/</sup> Here the concept of "retention rate" is used following the examples of Duncan (1965) and Stockwell and Nam (1963). However, the concept is used differently in the Office of Education's publications.

<sup>3/</sup> Treating educational system as an input-output matrix has been shown in OECD (1967), Ch. 2.

This approach has been used extensively in many countries for the projection of future school enrollments. (See, for example, Zabrowski et al., 1967; Thonstad, 1967). It is not the purpose of our paper to repeat the same effort; however, we find that this approach can serve as a powerful analytical device. We are going to show that a model of student progression can be generated from this approach.

Let us take the example of projecting the 2nd grade school population at time t+1. According to the Markov process principle expressed in Equation (2), it shall be:

(3) 
$$\eta_{(2)}^{t+1} = \eta_{(1)}^{t} \cdot A_{(1,2)} + \eta_{(2)}^{t} \cdot A_{(2,2)}$$

. . .

where  $A_{(1,2)}$  is the progression rate from the 1st grade to the 2nd grade;  $A_{(2,2)}$  is the failure rate for the 2nd grade students to remain in the same grade. What Equation (3) says is that the size of the student cohort at <u>x+1</u>th grade in the <u>t+1</u>th year is composed of those students of N(x,t) who can successfully progress to <u>x+1</u>th grade, and those students of N(x+1,t) who have to repeat in the same grade.

To extend this line of thinking, let us use some symbols to denote the following concepts:

- N(x,t) = the student population at the <u>x</u>th grade in the <u>t</u>th year.
- m(x) = the mortality rate of the <u>x</u>th grade.
- d(x) = the dropout rate at the <u>x</u>th grade.
- r(x) = the failure rate at the <u>x</u>th grade when students are supposed to progress to the <u>x+l</u>th grade.

Accordingly, a model can be generated from the aforementioned Markov-process consideration. The model, which is similar to that of equation (3), can be expressed as follows:

(4)  $N(x+1, t+1) = N(x, t) \cdot p(x) + N(x+1, t) \cdot r(x+1)$ 

Besides, it is seen from the transitional matrix that:

(5) 
$$p(x) = 1 - (m(x) + d(x) + r(x))$$

So the model in Equation (4) becomes:

(6) 
$$N(x+1,t+1) = N(x,t) \cdot (1 - (m(x) + d(x) + r(x))) + N(x+1,t) \cdot r(x+1)$$

#### A Conversion Technique

It appears that this model is not unique in the field of educational planning. As a matter of fact, most of the mathematical models of student progression are structurally more or less similar. However, they are different in the methods of implementing their models. The main effort of this paper, therefore, lies on the fact that we attempt to incorporate some of the existing demographic techniques into the implementation of our model.

Age is one of the basic demographic charac-teristics. It is "an invariant function of time and in this sense a fixed variable" (Duncan, 1965:123). So demographers are more inclined to measure social phenomenon by using age as an unit of measurement. This is very prevalent in the literature of educational demography. However, it is generally understood that the educational system is a structure of grade hierarchy. Consequently, the accomplishments of educational demographers are often ignored and their works are scarcely incorporated in the educational model-building. For instance, the work of Stockwell and Nam is cited by some educational planners (e.g., Correa, 1967; Dressel, 1967), and yet their ideas, particularly the measurement of dropout, have never been seriously employed. Here we attempt to show that some of the Stockwell and Nam's ideas can be readily applied to educational planning.

Stockwell and Nam's measurement of dropout is very similar to the measurement of retirement in labor force analysis (Wolfbein, 1949). It is a by-product of constructing the table of school life. First of all, there is a concept called "stationary school population," which is the result of multiplying age-specific enrollment rates by the stationary (life table) population. $\frac{4}{}$ Let us denote age as a, and the stationary school population at age a as N'(a). The ratio of the population at the successive ages, N'(a+1)/N'(a), shows the proportion of the life table population who remain in schools during a to a+l interval. The complement of this proportion gives the propensity that the population at age a will not be enrolled in schools at age a+1. The causes of this "separation factor" appear to be mortality and dropout. So dropout rate can be obtained by operationally differentiating mortality rate from this separation factor.

The mortality rate in a life table is simply the ratio of deaths to the life table population at a given age, where deaths are those in a population who do not survive from age a to age a+1. We shall denote the mortality rate as m(a). Then the propensity of dropout at age a is measured by: $\frac{5}{2}$ 

5/ The formula is slightly different from what Stockwell and Nam present. They have adjusted the denominator (population) by a half of deaths. To the present writer, the adjustment seems to be unnecessary; however, it is not the major concern of this paper to discuss such a technical difference.

<sup>4/</sup> For those who are not familiar with techniques of life table construction, a classical reference is: Dublin, L. I., A. J. Lotka, and M. Spiegelman, <u>Length of Life</u>, Ronald Press Co., N. Y., 1949.

(7) 
$$d(a) = 1 - N'(a+1)/N'(a) - m(a)$$

Table 2 presents the rates of dropout and mortality by age for the United States school population in 1960. The mortality rate is constructed by assuming that the risk of death in the school population is the same as the general population. This is perhaps not too far from the truth. The dropout rate is the residual of substrating the mortality rate from the total separation factor, as Equation (7) indicates. It is shown that the dropout rate increases from 2 per thousand at ages 10-11 to 350 per thousand at ages 18-19, and then it decreases, with fluctuations, to 120 per thousand at ages 29-30+.

It appears that an empirical finding such as this is very consistent with our general knowledge of student's progression in an educational system. For instance, the peak of dropping out of school occurs around age 18, which is approximately the time when students have completed high school. This observation raises a further inquiry, namely, to what extent the dropout rates in successive ages are corresponding to the dropout rate in a successive grade?

The question is also of practical importance, as it is generally observed that in this country different statistical agencies tend to use different classifications in publishing their educational data: some are age-specific, whereas others are grade-specific. For this reason, it is analytically necessary to obtain a transformation matrix which can show a correspondence between age and sex characteristics, so that a manipulation of the matrix will be able to convert any demographic characteristic from an age vector into a grade vector.

Let us denote a school population at age a of grade x as S(a,x). The total school population of all grades for age a will be  $\sum_{X} S(a,x)$ . It follows that there can be a matrix, T, with its elements as:

## TABLE 2

# MORTALITY AND DROPOUT AND SEPARATION RATES BY AGE UNITED STATES, 1960

	4. <u>1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1</u>		Separation Rate (per thousand)				
	Populat	Population			Enroll.		
Age	Life Table	School	Ratio	Total	Death	Dropout	
_		ha hha	1.1.0	0.7	07		
2	90,977	43,440	•440 ຊວນ	0.7	0.7	-	
0	90,913 06,860	00,025	.034	0.9	0.5	-	
	90,000	93,994	•970			-	
0	90,019	94,009	•970	0.4	0.4 0 h	-	
9	90,770	94,744	.900	2 h	0.4 0 L	2.0	
10	90,741	01 182	• 71 7 077	2.4 2 h	0.4	2.0	
12	90,101	oh 25h	.975	6.6	0.4	6.2	
12	96 631	93.635	.969	17.0	0.5	16.5	
14	96,583	92,044	.953	26.8	0.6	26.2	
15	96,526	89,576	.928	69.6	0.7	68.9	
16	96,457	83,339	.864	124.6	0.8	123.7	
17	96,377	72,957	•757	333.5	0.9	332.6	
18	96,287	48,625	.505	351.2	1.0	350.1	
19	96,189	31,550	.328	281.3	1.1	280.2	
20	96,085	22,676	.236	208.5	1.1	207.4	
21	95,975	17,947	.187	237.7	1.2	236.5	
22	95,859	11,887	.124	210.7	1.3	209.4	
23	95,739	9,382	.098	154.1	1.3	152.9	
24	95,618	7,936	.083	85.5	1.3	84.2	
25	95,496	7,258	.076	93.3	1.3	92.0	
26	95,374	6,581	.069	102.6	1.3	101.3	
27	95,252	5,906	.062	98.0	1.3	96.6	
28	95,128	5,327	.056	108.3	1.3	107.0	
29	95,002	4,750	.050	121.2	1.4	119.8	
30+	94,871	4,174	.044	-	-	-	

Source: United States Life Table, 1959-61, Public Health Service Publication No. 1252, Vol. 1, No. 1, Table 1, p. 8. United States Census of Population, 1960, Final Report PC(1)-1D,

Table 165, p. 371.

(8) 
$$T(a,x) = S(a,x) / \sum_{x} S(a,x)$$

It is possible, then, to treat the agespecific dropout rate, d(a), as a distribution vector, and postmultiply it to the matrix, T(a,x). The result gives a vector, d(x), which is the grade-specific dropout rate.



The same conversion technique can be employed to transform the age-specific mortality rate into the grade-specific mortality rate. An application of this measurement to the United States school population in 1960 is presented in Table 3. It shows the transformation matrix as well as the grade-specific dropout rate and mortality rate. The dropout rate appears to be a monotonously increasing function. It is less than 1 per thousand at the first grade, yet it increases sharply to about 300 per thousand at the end of the high school ages. On the other hand, the mortality rate is relatively constant. It is roughly about 5 per thousand, with a slight increase only after the 8th grade.

## Measurement of Failure Rate

In some countries where reliable repeater figures are available the implementation of an educational planning model will involve only a very simple manipulation. For example, Liu has used the repeater statistics in Colombia to demonstrate its utility in projecting future school population (Liu, 1966). In most of the countries, however, this kind of statistics is generally lacking. So the measurement of failure rate becomes a more or less arbitrary analytical exercise. Although in a study of Australian university enrollment, Gani has suggested that an arbitrary estimation of failure rate can also show an excellent projection of future school population (Gani, 1963), it seems more commmendable to pursue a more rigorous statistical approach.

In its DYNAMOD II model, the Office of Education has proposed a less arbitrary method of estimating failure rate. The technique is very much similar to a concept which demographers call "grade retardation" (See Folger and Nam, 1967: 8-11). For instance, in the estimate of the 1st grade's failure rate, it is taken as:

(9) 
$$r(1) = \frac{N(8, 1) + N(9, 1)}{N(5, 1) + N(6, 1) + N(7, 1) + N(8, 1) + N(9, 1)}$$

where N(a,x) is the school population at age a of grade x (Zabrowski and Hudman, 1967:3-4).

To what extent the measurement of retardation can represent the true picture of failure (repeater) rate remains an unanswered question, as there seems to be no empirical finding to verify such measurement so far. For this reason, we attempt to present another approach to measure the grade-specific failure rate. Although, it is not our contention that our measure is necessarily a more valid one, yet this alternative measurement may prove to be more consistent with the actual school-enrollment data.

Our approach is to make use of the model which is presented in Equation (6). The model can be changed by taking into account the measurement of separation rate as proposed in the last section. The separation rate is, by Stockwell and Nam's definition, m(x) + d(x), or mortality rate plus dropout rate. If we have the estimates of the separation rate, there can be a continuation rate, such as c(x) = 1 - (m(x) + d(x)). Then Equation (6) can be expressed as follows:

(10)  $N(x+1, t+1) = N(x, t) \cdot (c(x) - r(x)) + N(x+1, t) \cdot r(x+1)$ 

It is rearranged and so it becomes:

(11) 
$$N(x,t) \cdot r(x) - N(x+1,t) \cdot r(x+1)$$
  
=  $N(x,t) \cdot c(x) - N(x+1,t+1)$ 

where the parameters r(x) and r(x+1) are on the left side of the equation.

Let us assume that the mortality and dropout rates derived from the census data are applicable to the rest of the decade. By doing this, it is possible to implement the continuation rate c(x)for the equation. Furthermore, the size of the student population in each grade is given by the yearly school enrollment statistics, and so the implementation of N(x,t), N(x+1,t) or N(x+1,t+1)is also possible. Consequently, the only parameters left unsolved in Equation (11) are those failure rates, r(x) and r(x+1).

It is obvious that Equation (11) is recursive with respect to every grade. Therefore, the student progression in each grade can be expressed in one of the equations; r(x) can be r(1), r(2)... If we substitute the coefficients N(x,t) by  $\mathbf{A}$ , N(x+1,t) by  $\boldsymbol{\beta}$ , and  $N(x,t) \cdot c(x) - N(x+1,t+1)$ by  $\boldsymbol{\lambda}$ , the following recursive equations are obtained:

$$d_{1} r(1) - \beta_{1} r(2) = \lambda_{1}$$
  

$$d_{2} r(2) - \beta_{2} r(3) = \lambda_{2}$$
  

$$d_{3} r(3) - \beta_{3} r(4) = \lambda_{3}$$
  
....  

$$d_{11}r(11) - \beta_{11}r(12) = \lambda_{11}$$

#### TABLE 3

						Gr	ade					
	_ <b>I</b>	II	III	IV	<u>v</u>	VI	VII	VIII	IX	X	XI	XI
	.06	.01	-	-	-	-	_	-	-	-	-	-
	•57	.06	.01	-	-	-	-	-	-	-	-	-
	•32	•54	.06	.01	-	-	-	-	-	-	-	-
	.04	.32	•52	.04	.01	-	-	-	-	-	-	-
	.01	.06	•33	.51	.03	.01	-	-	-	-	-	-
	-	.01	.06	•33	.51	.04	.01	-	-	-	-	-
	-	-	.01	.07	•33	.51	.04	.01	-	-	-	-
	-	-	.01	.02	•08	•33	.51	.05	.01	-	-	-
	-	-	-	.01	.02	.08	•33	•54	•06	.01	-	-
	-	-	-	-	.01	.02	.07	.26	•53	.06	.01	-
	-	-	-	-	.01	.01	.03	•08	.30	.50	.06	•0
	-	-	-	-	-	-	.01	.03	.06	.30	•53	•0
	-	-	-	-	-	-	-	.01	.03	.07	.30	.4
	-	-	-	-	-	-	-	.01	.01	.02	.06	.2
	-	-	-	-	-	-	-	-	-	.01	.02	.0
	-	-	-	-	-	-	-	-	-	-	.01	.0
	-	-	-	-	-	-	-	-	-	-	-	.0
	-	-	-	-	-	-	-	-	-	-	-	.0
	-	-	-	-	-	-	-	-	-	-	. <b>-</b>	.0
	-	-	-	-	-	-	-	-	-	-	-	.0
	-	-	-	-	-	-	-	-	-	.01	.01	.0
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	.0009	.0018	.0028	.0031	.0050	.0086	.0170	.0344	.0800	.1126	.2000	.2
	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0005	.0006	.0007	.0008	.0

## DISTRIBUTION OF SCHOOL POPULATION BY AGE AND RATES OF DROPOUT AND MORTALITY FOR EACH GRADE UNITED STATES, 1960

Source: <u>United States Census of Population, 1960</u>, Final Report PC(1)-1D, Table 168, p. 377 and computed from Table 1.

Here the failure rates r(x), where x = 1,2,3..., are the only parameters. An application of these equations to the United States educational system will yield eleven equations. And there are twelve unknowns in these equations. The solution is possible only if one of the unknowns is given.

It is arbitrarily assumed that the failure rate at the first grade is 0.15; to some extent such a proportion is based on the retardation concept, as DYNAMOD II suggests. With this assumption, the solution of the recursive equations yields the values of grade-specific failure rate, r(x), where  $x = 1, 2, 3 \dots 11$ .

For illustration purpose, we apply this technique to the United States public school enrollment data from 1961 to 1965. Table 4 shows the estimates of failure rate at each level of education. It is found that the failure rate is relatively high in the very low grades. It declines to reach the lowest point at the 4th or 5th grade. Then it rises at the high school level.

If the quality of the data does not change much from year to year, the failure probabilities estimated through the above procedure will not show too much variation over time. Their stability contributes greatly to the reliability of the projection of future school population.

In a period from  $t_1$  to  $t_n$ , the solution of the recursive equations give  $r_t(x)$ , where x =1,2, ... ll and r = 1,2 ... n. An examination of their mean and variance is able to indicate if a stable pattern exists. If this is the case, a trend can be fixed to these values; for instance, the trend can be a linear equation such as  $r_t(x) =$ a + bt. The projected value of r(x) at  $t_{n+1}$  will not differ much from the average value if the variance is not too large.

Table 4 also shows the mean, variance and the projected values of 1965-66 failure rate. Considering the fact that the quality of the data are not very satisfactory, it is interesting to note that the results do not show any particularly great deviation from year to year. As a matter of fact, the failure rates under the 7th grade are very consistent. Their variances are negligible, and after the nth grade the variance increases only slightly. Consequently, the projected failure rate, as expected, is found to be not much different from the average value of the

Grade	1961-62	1962-63	1963 <b>-</b> 64	1964 <b>-</b> 65	Average	Variance	Projected 1965 <b>-</b> 66
1-2	.1500	.1500	.1500	.1500	.1500	.0	.1500
2-3	.0996	.1022	.1037	.1003	.1014	.0	.1023
3-4	.0908	.0872	.0973	.0880	.0908	.0	.0912
<b>4-</b> 5	.0856	.0792	.0927	.0810	.0845	.0	.0846
5-6	.0896	.0773	.0990	.0801	.0865	.0001	.0848
6-7	.0892	.0749	.1038	.0739	.0855	.0001	.0812
7-8	.1177	.0990	.1312	.0980	.1115	.0002	.1048
8-9	.1203	.0930	.1397	.0999	.1133	.0003	.1064
9-10	.1770	.1424	.1854	.1569	.1653	.0003	.1611
10-11	.2029	.1504	.2054	.1793	.1843	.0006	.1806
11-12	.2036	.1706	.1921	.1694	.1840	.0002	.1637

FAILURE RATES BY GRADE IN THE UNITED STATES PUBLIC SCHOOLS, 1961-1966

Source: Computed from <u>Digest of Educational Statistics</u>, 1966, Table 28, "Enrollment by Grade in Full-time Public Elementary and Secondary Day Schools," p. 24.

1961-64 period. The deviation is especially low in the elementary schools. So the result seems to indicate that there is a stable pattern of failure rates from year to year.

It appears that this technique of estimating the future failure rate may be acceptable. Therefore, the projected failure rate, along with the mortality and dropout rates, can be used in the projection of future school population. The projection procedure will be simply the application of the cohort approach as presented in Equation (6).

## Summary and Discussion

This paper treats the size of student cohort as affected by three components, namely, the previous cohort's mortality, dropout, and failure. Implicitly it assumes that the educational system is a closed population. So it follows that a model of student progression can be implemented using the renewal theories in demographic analysis.

The major part of this paper attempts to assess the components of the model. It is in this aspect that this model differs from some other models of educational planning: We propose a conversion technique to obtain the gradespecific dropout and mortality rates, and we also suggest that the grade-specific failure rate can be estimated by solving the recursive equations. Through extending this type of analysis, it is possible to reach a more realistic projection of school population.

Nevertheless, this paper has not attempted to elaborate in detail all the possible approaches for reaching a successful projection of school population. For instance, one area which has been overlooked is that of stratification of the population. Males and females, whites and nonwhites, have significantly different educational parameters that change at different rates over time. Consequently, any effort to produce a useful model for other than very short-range projections must take into account the population strata.

We also conclude that a future effort shall be taken to limit the assumption that the educational system is a closed population. Conceivably this assumption may not be unrealistic in some countries where dropouts have very little chance of re-entering school; however, it is too strenuous to be applicable to the United States public school system. A study by the Bureau of Labor Statistics (1966) shows that there is a considerable number of dropouts who return to school. Therefore, we expect a future development of this paper shall be able to treat school population as an open system, and so migration and re-enrollment can be taken into account.

## References

- Correa, Hector. 1967. "A Survey of Mathematical Models in Educational Planning," <u>Mathematical</u> <u>Models in Educational Planning</u>, Paris: Organization for Economic Cooperation and Development.
- Dressel, P. L. 1967. "Comments on the Use of Mathematical Models in Educational Planning," <u>Mathematical Models in Educational Planning</u>, Paris: Organization for Economic Cooperation and Development.

- Duncan, Beverly. 1965. "Dropouts and the Unemployed," The Journal of Political Economy, 73:121-134.
- Gani, J. 1963. "Formulae for Projecting Enrollments and Degrees Awarded in Universities," <u>Journal of the Royal Statistical Society</u>, Series A, 126:400-409.
- Folger, J. K. and C. B. Nam. 1967. Education of the American Population, Washington, D. C.: Bureau of the Census.
- Liu, B. A. 1966. <u>Estimating Future School</u> <u>Enrollment in Developing Countries</u>, N. Y.: UNESCO, Population Studies No. 40.
- Moser, C. A. and P. Redfern. 1965. "A Computable Model of the Educational System in England and Wales," <u>Proceeding of the Inter-</u> <u>national Institute of Statistics</u>, Belgrade, 1965:693-700.
- Organization for Economic Cooperation and Development. 1967. <u>Methods and Statistical</u> <u>Needs for Educational Planning</u>, Paris: OECD.
- Pressat, Roland. 1961. <u>L'analyse Demographique</u>, Paris: Preses Universitaires de France.
- Siegel, Jacob S. 1967. "Revised Projections of School and College Enrollment in the United States to 1985," <u>Current Population Reports</u>, Series P-25, No. 365.
- Simon, K. A. and W. V. Grant. 1965. <u>Digest of</u> <u>Educational Statistics</u>, 1965 edition, Washington, D. C.: Office of Education.
- Stockwell, E. G. and C. B. Nam. 1963. "Illustrative Tables of School Life," <u>Journal of</u> <u>the American Statistical Association</u>, 58: <u>1113-1124</u>.

- Stone, Richard. 1966. "A Model of the Educational System," <u>Mathematics in the Social</u> <u>Sciences and Other Essays</u>, Cambridge: The <u>M.I.T. Press, Ch. IX</u>, 104-117.
- Thonstad, Tore. 1967. "A Mathematical Model of the Norwegian Educational System," <u>Mathe-</u> <u>matical Models in Educational Planning</u>, Paris: Organization for Economic Cooperation and Development.
- U. S. Bureau of Census. 1966. "Projections of School and College Enrollment in the United States to 1985," <u>Current Population Reports</u>, P-25, No. 388.
- U. S. Bureau of Labor Statistics. 1966. <u>Out-of-</u> <u>School Youth-sTwo Years Later</u>, Special Labor Force Report No. 71.
- Wolfbein, S. L. 1949. "The Length of Working Life," <u>Population Studies</u>, 3:286-294.
- Wurtele, Zivia S. 1967. <u>Mathematical Models for</u> <u>Educational Planning</u>, System Development Corporation, SP-3015.
- Zabrowski, E. K. and J. T. Hudman. 1%7. <u>Dropout and Retention Rate Methodology Used to</u> <u>Estimate First-Stage Elements of the Transition Probability Matrices for DYNAMOD II,</u> Technical Note 28, National Center for Educational Statistics.
- Zabrowski, E. K., J. T. Hudman, T. Okada and J. R. Zinter. 1967. <u>Student-Teacher Popula-</u> <u>tion Growth Model: DYNAMOD II</u>, Technical Note 34, National Center of Educational Statistics.